Example 1: Find $\frac{d^2y}{dx^2}$

if

$$\sin(xy^2) = \frac{1}{2}.$$

Differentiate both sides with respect to x.

We get $\longrightarrow$

$$\cos(xy^2)\frac{d}{dx}(xy^2)=\frac{d}{dx}\left(\frac{1}{2}\right)=0$$

$$\cos(xy^2)\left(y^2+x\frac{d}{dx}(y^2)\right)=0$$

$$\cos(xy^2)\left(y^2+2xy\frac{dy}{dx}\right)=0$$

$$y^2\cos(xy^2)+2xy\cos(xy^2)\frac{dy}{dx}=0$$

Subtract $y^2\cos(xy^2)$

from both sides

$$2yx\cos(xy^2)\frac{dy}{dx} = -y^2\cos(xy^2)$$

Divide both sides by

$$2yx\cos(xy^2)$$

$$\frac{dy}{dx} = \frac{-y^2\cos(xy^2)}{2yx\cos(xy^2)}$$

$$= \frac{-y^2}{2yx} = \frac{-y}{2x}$$

Differentiate again...

$$\frac{dy}{dx} = -\frac{1}{2} y \cdot x^{-1}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{2} \frac{d}{dx}\left(y \cdot x^{-1}\right)$$

$$= -\frac{1}{2}\left(\frac{dy}{dx} \cdot x^{-1} - x^{-2} y\right)$$

$$= -\frac{1}{2}\left(\frac{dy}{dx} \cdot \frac{1}{x} - \frac{y}{x^2}\right)$$

$$= -\frac{1}{2}\left(-\frac{1}{2}\frac{y}{x^2} - \frac{y}{x^2}\right)$$

$$= \boxed{3y/4x^2}$$

Related Rates (section 2.8)

All story problems, always with implicit differentiation (usually with respect to time)

Main Things You'll Use

0) Volumes of familiar objects (sphere, cube, cone, etc)

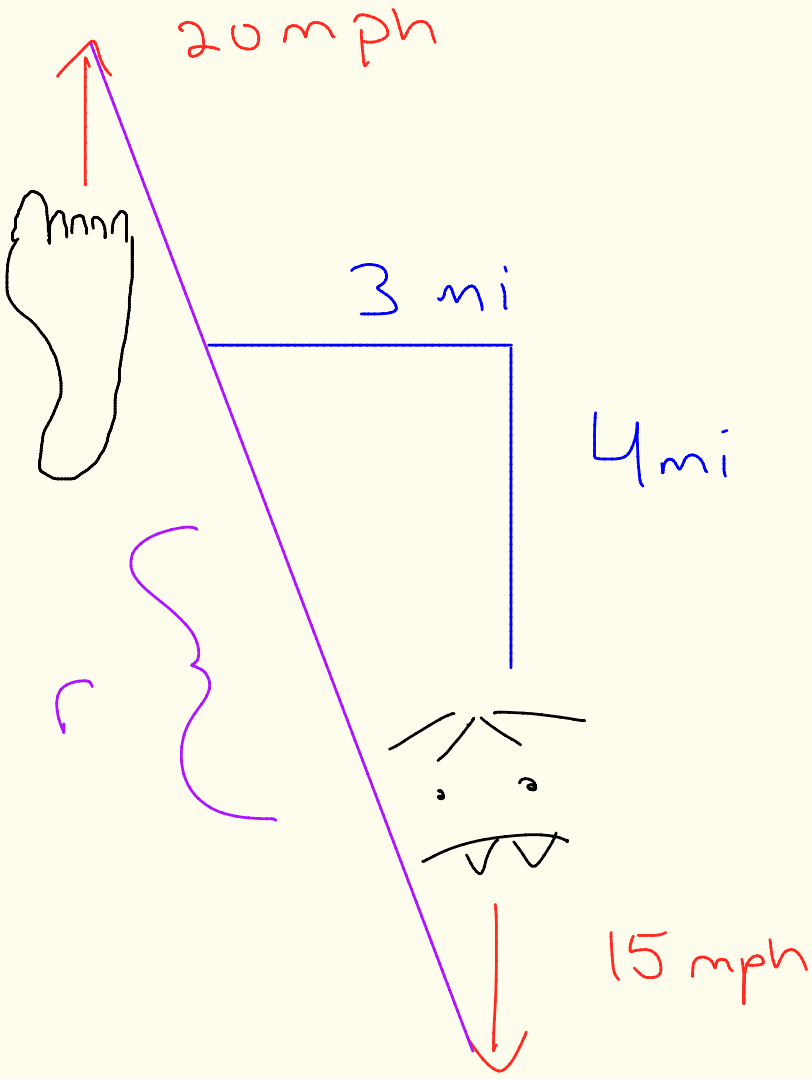
1) Pythagorean Theorem / Distance Formula

2) Definitions of Trig Functions

3) Similar Triangles

Example 2: Bigfoot

is walking north at a rate of 20 miles per hour. At the same time, Sasquatch is walking south at 15 miles per hour. If Sasquatch is 3 miles east and 4 miles south of Bigfoot at 12 noon, how fast is the distance between them changing at 3 PM?



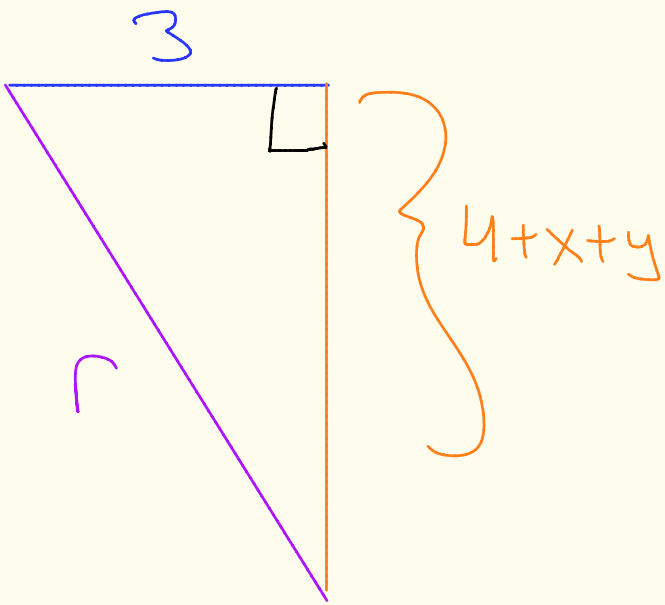
r = distance between Bigfoot and Sasquatch

Let x = distance walked by Bigfoot after noon.

Let y = distance walked by Sasquatch after noon

Know: $\frac{dx}{dt} = 20, \frac{dy}{dt} = 15$

Find a relationship between $x, y,$ and r.



So $(4+x+y)^2 + 3^2 = r^2$

$(4+x+y)^2 + 9 = r^2$

Differentiate both sides of $(4+x+y)^2+9=r^2$ with respect to t:

$$2(4+x+y)\left(\frac{dx}{dt}+\frac{dy}{dt}\right)=2r\frac{dr}{dt}$$

Finish next class